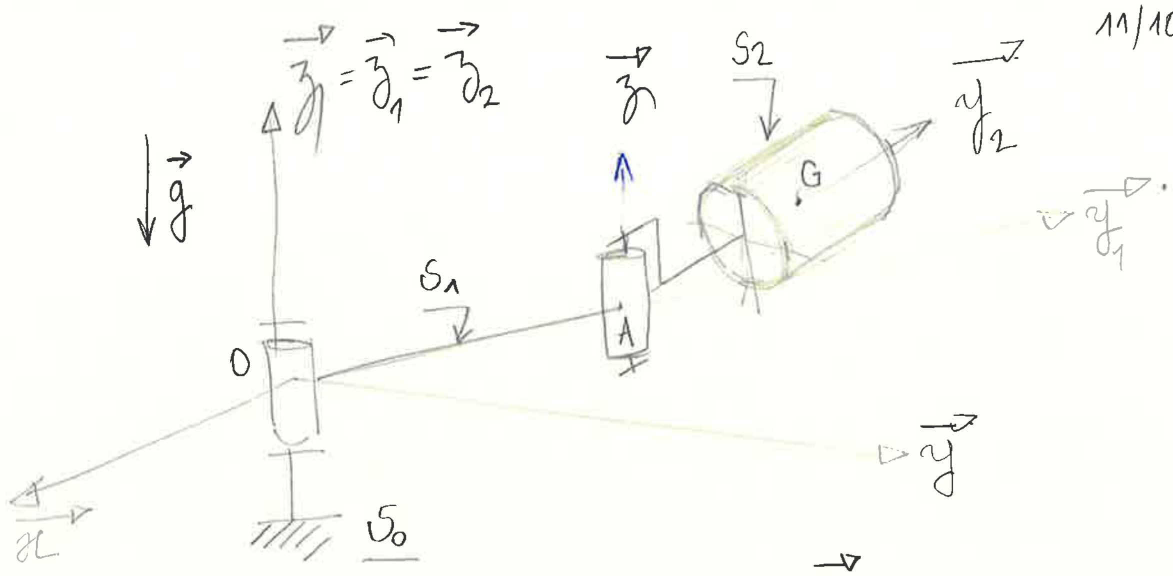
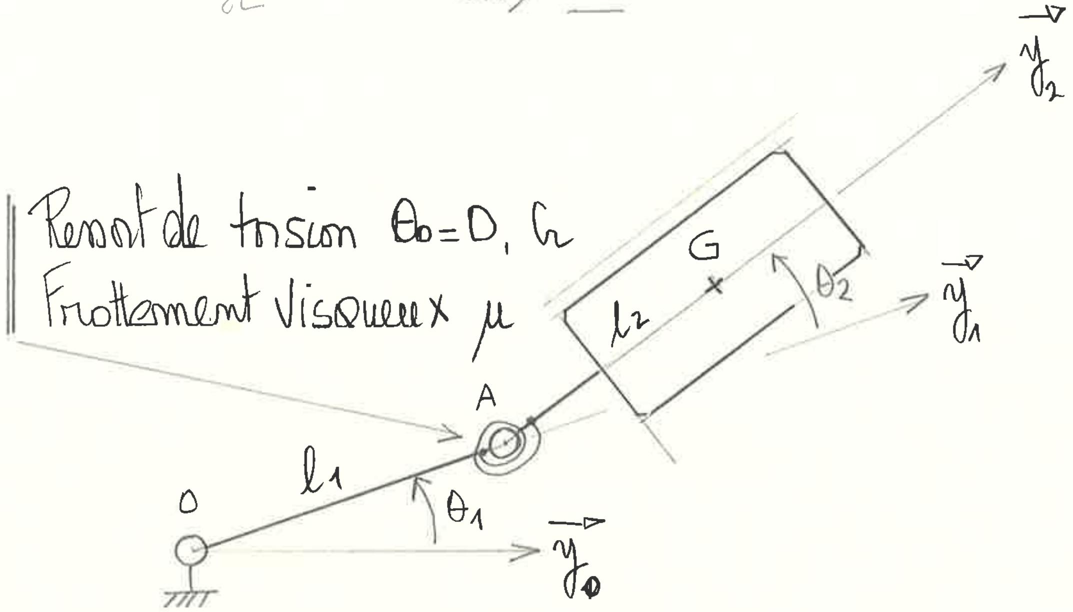
11/10/2019





$$\bar{\bar{I}}_G(S_2) = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A \end{vmatrix}_{G,\, b_2}$$

C_r : Nm/rd.

μ : Nm/rd·s^{-1}

Comment évolue θ_2, $\dot{\theta}_2$, $\ddot{\theta}_2$ en fθ de θ_1, $\dot{\theta}_1$, $\ddot{\theta}_2$?

θ_2 pourra être considéré petit ainsi que $\dot{\theta}_1$ et $\dot{\theta}_2$

1) On cherche une équation de mouvement.

$\Rightarrow$ Théorème . projection.

Concerne une rotation, un angle, pivot en A

$\Rightarrow$ Théorème moment dynamique projeté sur l'axe de rotation

$$\sum \overrightarrow{Mt_A}\ \overrightarrow{Fext} \rightarrow 2 \cdot \vec{z_1} = \overrightarrow{\delta_A}(2/Rg) \cdot \vec{z_1}$$

On isole, en effet, 2.

2) Bilan des actions $\rightarrow S_2$.

$$\left\{\mathcal{F}_{g \rightarrow 2}\right\} = \left\{-mg\,\vec{z_1} ; \vec{0}\right\}_G$$

$$\left\{\mathcal{F}_{1 \rightarrow 2}\right\} = \left\{XA_{12}\,\vec{x_1} + YA_{12}\,\vec{y_1} + ZA_{12}\,\vec{z_1} ; LA_{12}\,\vec{x_1} + MA_{12}\,\vec{y_1} + 0\,\vec{z_1}\right\}_A$$

$$\left\{\mathcal{F}_{ressort \rightarrow 2}\right\} = \left\{\vec{0} ; -C_r\,\theta_2\,\vec{z_1}\right\}$$

$$\left\{\mathcal{F}_{frott \rightarrow 2}\right\} = \left\{\vec{0} ; -\mu\,\dot{\theta}_2\,\vec{z_1}\right\}$$

$$\Rightarrow \sum \overrightarrow{Mt_A}\ \overrightarrow{Fext} \rightarrow 2 \cdot \vec{z_1} = -C_r\,\theta_2 - \mu\,\dot{\theta}_2$$

3) Moment dynamique :

$$\begin{tikzcd}
\overrightarrow{\delta_A}(2/Rg) & \overrightarrow{\sigma_A}(2/Rg) \arrow[l, "\text{Dérivation}"'] & \overrightarrow{\sigma_G}(2/Rg) \arrow[l, "\text{Transport}"'] \arrow[dll, "D"] \\
\overrightarrow{\delta_G}(2/Rg) \arrow[u, "T"'] & &
\end{tikzcd}$$

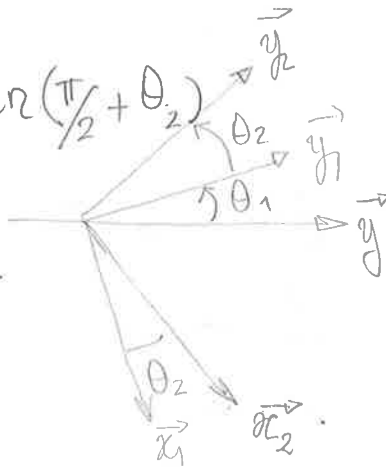
$$\overrightarrow{\sigma_G}(2/Rg) = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A \end{vmatrix}_{G,2} \cdot \begin{vmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{vmatrix}_2 = A(\dot{\theta}_1 + \dot{\theta}_2)\,\vec{z_1}$$

(La matrice est en G)

$$\vec{\sigma}_A(2/R_g) = \vec{\sigma}_G(2/R_g) + \vec{AG} \wedge m\, \vec{V}_{G/R_g}$$

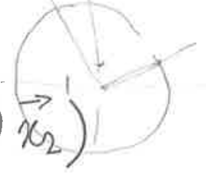
$$= A(\dot\theta_1+\dot\theta_2)\vec{z} + l_2\vec{y}_2 \wedge m\left(-l_1\dot\theta_1\vec{x}_1 - l_2(\dot\theta_1+\dot\theta_2)\vec{x}_2\right)$$

$$= (A+ml_2^2)(\dot\theta_1+\dot\theta_2)\vec{z} + m\, l_1 l_2\, \dot\theta_1 \sin(\pi/2+\theta_2)$$



$$= (A+ml_2^2)(\dot\theta_1+\dot\theta_2)\vec{z} - m\, l_1 l_2\dot\theta_1 \cos\theta_2 .$$

$$\vec{\delta}_A(2/R_g) = \left[\frac{d}{dt}\vec{\sigma}_A(2/R_g)\right]_{R_g} + m\,\vec{V}_{A/R_g} \wedge \vec{V}_{G/R_g}$$



$$+\left(-m\, l_1\dot\theta_1\vec{x}_1\right) \wedge \left(-l_1\dot\theta_1\vec{x}_1 - l_2(\dot\theta_1+\dot\theta_2)\vec{x}_2\right)$$

$$\vec{\delta}_A(2/R_g)\cdot\vec{z} = (A+ml_2^2)(\ddot\theta_1+\ddot\theta_2) - m\,l_1 l_2\ddot\theta_1 \underset{1}{\cancel{\cos\theta_2}} + m\,l_1 l_2\dot\theta_1\dot\theta_2 \underbrace{\sin\theta_2}_{\theta_2}$$

$$+ m\, l_1 l_2\, \dot\theta_1(\dot\theta_1+\dot\theta_2)\underbrace{\sin\theta_2}_{\theta_2}$$

$$\Rightarrow -C\theta_2 - \mu\dot\theta_2 = (A+ml_2^2)(\ddot\theta_1+\ddot\theta_2) - m\,l_1 l_2\ddot\theta_1 + m\,l_1 l_2\dot\theta_1\dot\theta_2\theta_2 + m l_1 l_2\dot\theta_1(\dot\theta_1+\dot\theta_2)\theta_2$$

si $\dot\theta_1$ et $\dot\theta_2$ petits et θ_2 petit

$$(A+ml_2^2)\ddot\theta_2 + \mu\dot\theta_2 + C\,\theta_2 = (A+ml_2^2 - m\,l_1 l_2)\ddot\theta_1$$