

de torsion $\Theta_0 = 0$, ζ
ment visqueux μ

The diagram shows a mechanical system with two rotating bodies. Body 1 is a horizontal rod of length l_1 pivoted at point O. It has a coordinate system (y_0, z_0) and an angular displacement θ_1 . Body 2 is a rectangular plate of length l_2 pivoted at point A. It has a coordinate system (y_1, z_1) and an angular displacement θ_2 . The rod is connected to the plate at point A. The system is subject to a torsional spring at O and a viscous damper at A.

$$\mathbb{I}_G(S_2) = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A \end{vmatrix}_G \quad \begin{matrix} \\ \\ b_2 \end{matrix}$$

Cr: Nm/rd.

$$\mu: \text{Nm} / \text{rad s}^{-1}$$

Comment évolue $\theta_2, \dot{\theta}_2, \ddot{\theta}_2$ en f^{on} de $\theta_1, \dot{\theta}_1, \ddot{\theta}_1$?

θ_2 pourra être considéré petit ainsi que $\dot{\theta}_1$ et $\dot{\theta}_2$

1) On cherche une équation de mouvement.

⇒ Théorème : projection.

Concerne une rotation, un angle, pivot en A

⇒ Théorème moment dynamique projeté sur l'axe de rotation

$$\sum \overrightarrow{Mto_A} \overrightarrow{F_{ext} \rightarrow 2} \cdot \vec{z} = \overrightarrow{\sigma_A(2/R_g)} \cdot \vec{z}$$

On isole, en ekr, 2.

2) Bilan des actions → S2.

$$\{\vec{F}_{g \rightarrow 2}\} = \{-mg \vec{z}; \vec{0}\}_G$$

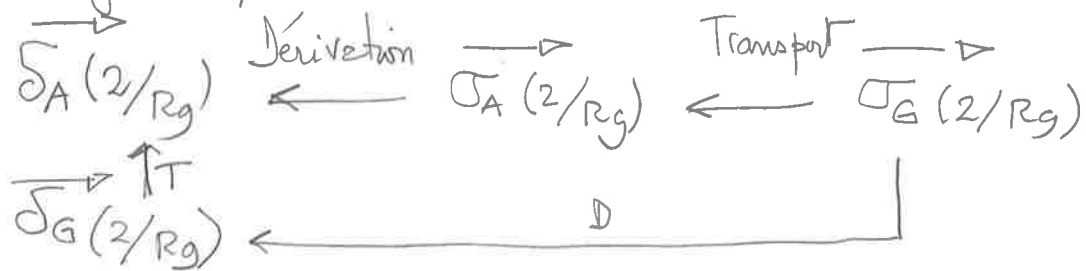
$$\{\vec{F}_{1 \rightarrow 2}\} = \{X_{A12} \vec{x}_1 + Y_{A12} \vec{y}_1 + Z_{A12} \vec{z}; L_{A12} \vec{x}_1 + M_{A12} \vec{y}_1 + 0 \vec{z}\}_A$$

$$\{\vec{F}_{ressort \rightarrow 2}\} = \{\vec{0}; -C \theta_2 \vec{z}\}$$

$$\{\vec{F}_{frott \rightarrow 2}\} = \{\vec{0}; -\mu \dot{\theta}_2 \vec{z}\}$$

$$\Rightarrow \sum \overrightarrow{Mto_A} \overrightarrow{F_{ext} \rightarrow 2} \cdot \vec{z} = -C \theta_2 - \mu \dot{\theta}_2$$

3) Moment dynamique :



$$\overrightarrow{\sigma_G(2/R_g)} = \begin{vmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A \end{vmatrix}_G \cdot \begin{vmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{vmatrix} = A(\dot{\theta}_1 + \dot{\theta}_2) \vec{z}$$

(la matrice est)

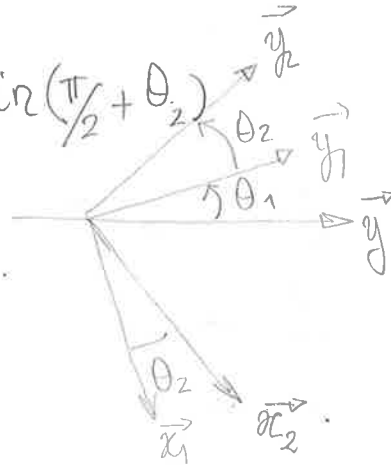
en G

$$\vec{\sigma}_A(z/R_g) = \vec{\sigma}_G(z/R_g) + \vec{AG} \wedge m \cdot \vec{V}_{G/R_g}$$

$$= A(\dot{\theta}_1 + \dot{\theta}_2) \vec{z} + l_2 \vec{y}_2 \wedge m(-l_1 \dot{\theta}_1 \vec{x}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2) \vec{x}_2)$$

$$= (A + m l_2^2)(\dot{\theta}_1 + \dot{\theta}_2) \vec{z} + m l_1 l_2 \dot{\theta}_1 \sin(\pi/2 + \theta_2) \vec{y}_2$$

$$= (A + m l_2^2)(\dot{\theta}_1 + \dot{\theta}_2) \vec{z} - m l_1 l_2 \dot{\theta}_1 \cos \theta_2$$



$$\vec{\sigma}_A(z/R_g) = \left[\frac{d}{dt} \vec{\sigma}_A(z/R_g) \right]_{R_g} + m \vec{V}_{A/R_g} \wedge \vec{V}_{G/R_g}$$

$$+ (-m l_1 \dot{\theta}_1 \vec{x}_1) \wedge (-l_1 \dot{\theta}_1 \vec{x}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2) \vec{x}_2)$$



$$\vec{\sigma}_A(z/R_g) \cdot \vec{z} = (A + m l_2^2)(\ddot{\theta}_1 + \ddot{\theta}_2) - m l_1 l_2 \ddot{\theta}_1 \cos \theta_2 + m l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin \theta_2$$

θ_2

$$+ m l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2$$

θ_2

$$\Rightarrow -G \theta_2 - \mu \dot{\theta}_2 = (A + m l_2^2)(\ddot{\theta}_1 + \ddot{\theta}_2) - m l_1 l_2 \ddot{\theta}_1 + m l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \theta_2 + m l_1 l_2 \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2) \theta_2$$

si $\dot{\theta}_1$ et $\dot{\theta}_2$ petits et θ_2 petit

$$(A + m l_2^2) \ddot{\theta}_2 + \mu \dot{\theta}_2 + G \theta_2 = (A + m l_2^2 - m l_1 l_2) \ddot{\theta}_1$$