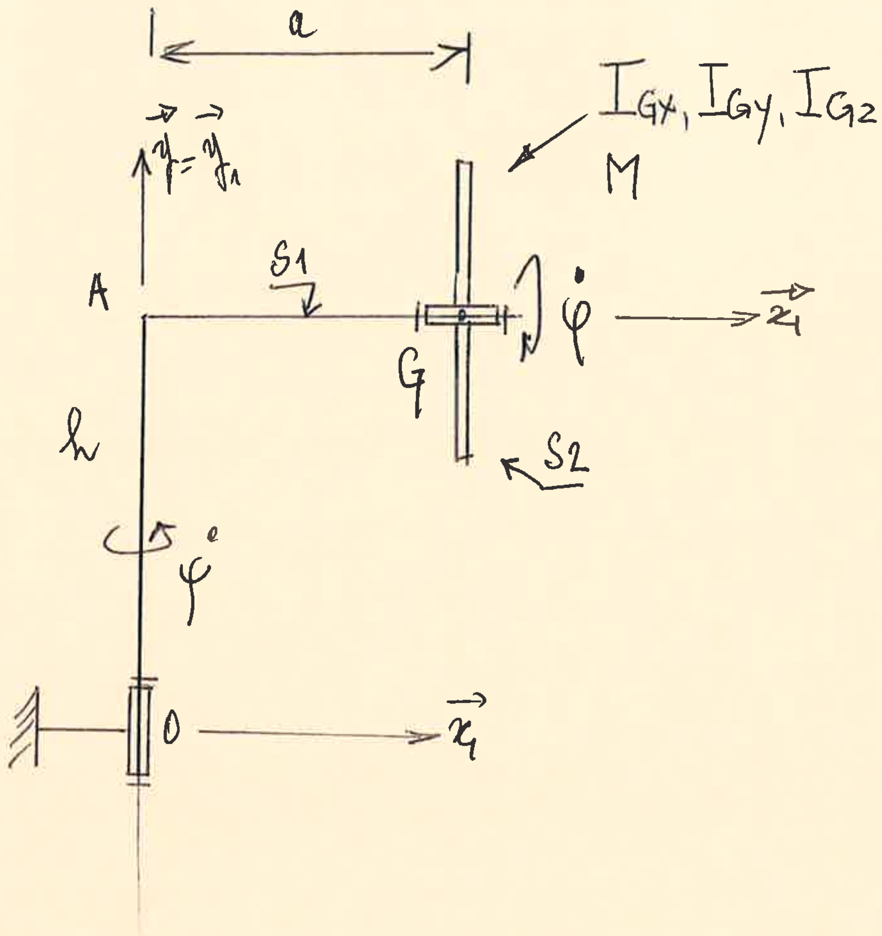


a
I_{Gx}, I_{Gy}, I_{Gz}
M
$\vec{y} = \vec{y}_1$
A
S1
$\dot{\varphi}$
$\vec{z}_1$
G
h
S2
$\dot{\psi}$
0
$\vec{x}_1$

On isole $S_1 + S_2$.

Bilan.

$$_{b_1}\left\{\mathcal{F}_{0\to 1}\right\} = \left\{X0_{01}\vec{x}_1 + Y0_{01}\vec{y}_1 + Z0_{01}\vec{z}_1 \,;\, L0_{01}\vec{x}_1 + 0\,\vec{y}_1 + N0_{01}\vec{z}_1\right\}_0$$

$$\left\{\mathcal{F}_{g\to 2}\right\} = \left\{-Mg\,\vec{y}_1 \,;\, -Mga\,\vec{z}_1\right\}_0$$

$$\left\{\mathcal{C}\,2/R_g\right\} = \left\{-a\dot{\psi}M\vec{z}_1 \,;\, \overrightarrow{\sigma_G}(2/R_g)\right\}_G$$

$$\overrightarrow{\sigma_G}(2/R_g) = \begin{vmatrix} I_{Gx} & 0 & 0 \\ 0 & I_{Gy} & 0 \\ 0 & 0 & I_{Gz} \end{vmatrix}_{b_1} \begin{vmatrix} \dot{\varphi} \\ \dot{\psi} \\ 0 \end{vmatrix}_{b_1} + 0 = \begin{vmatrix} I_{Gx}\dot{\varphi} \\ I_{Gy}\dot{\psi} \\ 0 \end{vmatrix}_{b_1}$$

$$\left\{\mathcal{C}\,2/R_g\right\} = \left\{-Ma\dot{\psi}\vec{z}_1 \,;\, I_{Gx}\dot{\varphi}\vec{x}_1 + I_{Gy}\dot{\psi}\vec{y}_1\right\}_G.$$

$$\left\{\mathcal{C}\,2/R_g\right\}_0 = \left\{-Ma\dot{\psi}\vec{z}_1 \,;\, \left[I_{Gx}\dot{\varphi} - Mha\dot{\psi}\right]\vec{x}_1 + \left[I_{Gy} + Ma^2\right]\dot{\psi}\vec{y}_1\right\}_0$$

$$\overrightarrow{\sigma_0}(S/R_g) = \overrightarrow{\sigma_G}(S/R_g) + \overrightarrow{OG} \wedge M\,\overrightarrow{V_{G/R_g}}.$$

$$\begin{vmatrix} I_{Gx}\dot{\varphi} \\ I_{Gy}\dot{\psi} \\ 0 \end{vmatrix}_{b_1} + \begin{vmatrix} a \\ h \\ 0 \end{vmatrix}_{b_1} \wedge M \begin{vmatrix} 0 \\ 0 \\ -a\dot{\psi} \end{vmatrix}_{b_1}$$

$$\{\mathcal{D}(S/R_g)\} = \left\{ \underbrace{\cancel{-Ma\ddot{\psi}\,\vec{z}_1}}_{=0} - Ma\dot{\psi}^2\,\vec{x}_1 \;;\; \overrightarrow{\delta_O(S/R_g)} \right\}$$

O point fixe

$$\overrightarrow{\delta_O(S/R_g)} = \left[\frac{d}{dt}\overrightarrow{\sigma_O(S/R_g)}\right]_1 + \vec{\Omega}_{1/R_g} \wedge \overrightarrow{\sigma_O(S/R_g)} + \vec{0}$$

$$\overrightarrow{\delta_O(S/R_g)} = {}_1\begin{vmatrix} 0 \\ \dot{\psi} \\ 0 \end{vmatrix} \wedge {}_1\begin{vmatrix} I_{Gx}\,\dot{\varphi} - Mha\,\dot{\psi} \\ (I_{Gy} + Ma^2)\,\dot{\psi} \\ 0 \end{vmatrix}$$

$$= {}_{b_1}\begin{vmatrix} 0 \\ 0 \\ -I_{Gx}\,\dot{\psi}\dot{\varphi} + Mha\,\dot{\psi}^2 \end{vmatrix}$$

si $a = 0$

$$N_{O01} = \quad - I_{Gx}\,\dot{\varphi}\dot{\psi})$$

$$\Rightarrow \quad N_{O01} = -I_{Gx}\,\dot{\varphi}\dot{\psi}.$$

Couple gyroscopique.

On peut montrer, dans le cas général d'une toupie, que le couple gyroscopique $\vec{C}_{gyro}$:

$$\vec{C}_{gyro} = -I_{G\vec{u}} \cdot \dot{\varphi} \cdot \left[\frac{d}{dt}\vec{u}\right]_{R_g}$$

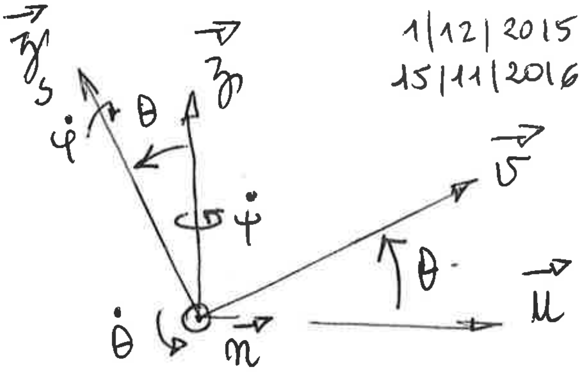
ici $\vec{C}_{gyro} = - I_{Gx} \cdot \dot{\varphi} \cdot \left[\frac{d}{dt} \vec{x_1} \right]$

$\vec{C}_{gyro} = - I_{Gx} \cdot \dot{\varphi} \, \dot{\psi} \cdot \vec{y_1}$.

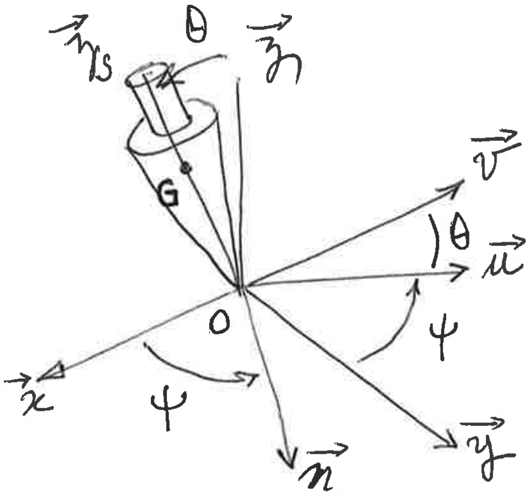
1) les angles d'Euler

ψ, θ, φ.

1/12/2015
15/11/2016



2) La toupie.



$$\begin{vmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{vmatrix}_{(-,-,\vec{z}_s)}$$

$$\vec{\Omega}_{S/R_g} = \begin{vmatrix} \dot\theta \\ \dot\psi \sin\theta \\ \dot\psi\cos\theta + \dot\varphi \end{vmatrix}_{(n\,v\,z_s)}$$

$$\dot\psi \vec{z} = \dot\psi(\cos\theta\,\vec{z}_s + \sin\theta\,\vec{v})$$

<u>O fixe.</u>

$$\vec{\sigma}_O = \begin{vmatrix} A\dot\theta \\ A\dot\psi\sin\theta \\ C(\dot\psi\cos\theta + \dot\varphi) \end{vmatrix}_{(n v z_s)}$$

base v $(\vec{n}\vec{v}\vec{z}_s)$

$\vec{\Omega}_{v/R_g}$

$$\vec{\delta}_O = \begin{vmatrix} A\ddot\theta \\ A\ddot\psi\sin\theta + A\dot\psi\dot\theta\cos\theta \\ C\ddot\psi\cos\theta - C\dot\psi\dot\theta\sin\theta + C\ddot\varphi \end{vmatrix}_{n v z_s} + \begin{vmatrix} \dot\theta \\ \dot\psi\sin\theta \\ \dot\psi\cos\theta + \cancel{\dot\varphi} \end{vmatrix}_{n v z_s} \wedge \begin{vmatrix} A\dot\theta \\ A\dot\psi\sin\theta \\ C(\dot\psi\cos\theta + \dot\varphi) \end{vmatrix}_{n v z}.$$

!

1|12|2015
15|11|2016

$$\vec{\delta_O} = \begin{vmatrix} \cancel{A\ddot{\theta}} + \cancel{C\dot{\psi}^2 \sin\theta\cos\theta} + \boxed{C\dot{\psi}\dot{\varphi}\sin\theta} - \cancel{A\dot{\psi}^2 \sin\theta\cos\theta} \\ A\ddot{\psi}\sin\theta + A\dot{\psi}\dot{\theta}\cos\theta - C\dot{\psi}\dot{\theta}\cos\theta - C\dot{\varphi}\dot{\theta} + A\dot{\theta}\dot{\psi}\cos\theta \\ \cancel{C\ddot{\psi}\cos\theta - C\dot{\psi}\dot{\theta}\sin\theta + C\ddot{\varphi}} + A\cancel{\dot{\theta}\dot{\psi}\sin\theta} - A\dot{\theta}\cancel{\dot{\psi}\sin\theta} \end{vmatrix}$$

$\dot{\varphi} = \underline{\underline{cte}}$ $\quad \theta = cte \quad \dot{\psi} = cte \quad \dot{\psi}\cos\theta \ll \dot{\varphi}$

$$\vec{\mathcal{M}}_O = \vec{OG} \wedge -mg\vec{z} = L\vec{z_s} \wedge -mg\vec{z} = Lmg\sin\theta\,\vec{n}$$

$$\vec{\mathcal{M}}_O \cdot \vec{n} = Lmg\sin\theta$$

$$\vec{\delta_O} \cdot \vec{n} = -C\,\dot{\psi}\dot{\varphi}\sin\theta$$ couple gyroscopique

$$\boxed{\dot{\psi} = \frac{-Lmg\cancel{\sin\theta}}{C\,\dot{\varphi}\cancel{\sin\theta}}}$$

$$\dot{\psi} = \frac{-0{,}02 \cdot 0{,}05 \cdot 10 \cdot 60}{C \cdot 1000 \times 2\pi}$$

<u>Couple gyroscopique</u>

$$-C\dot{\varphi}\left[\frac{d\vec{z_s}}{dt}\right]_0 = -C\dot{\varphi}\begin{vmatrix}\dot{\theta} \\ \dot{\psi}\sin\theta \\ \dot{\psi}\cos\theta\end{vmatrix} \wedge \begin{vmatrix}0 \\ 0 \\ 1\end{vmatrix} = -C\dot{\varphi}\dot{\psi}\sin\theta\,\vec{n}$$